The Simple Pendulum

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Introduction:

The simple pendulum is a mass hanging from the end of a string which is connected above in such a way to allow the mass to swing freely. The period, P, is the time it takes for the mass to swing from it's initial amplitude, across the vertical line though it's connecting point, up to a point of nearly the same amplitude but opposite the starting point, and finally back to (nearly) the original starting point. This action is called an oscillation. The period is given by

$$P=2\pi\sqrt{(L/g)}\cdot(1+1/4\sin^2(A)+9/64\sin^4(A/2)+...)$$

Where P is the period, A is the amplitude (angle), g is acceleration due to gravity (980 cm/sec^2), and L is the length. If the amplitude is small then $\sin^2(A)$ is small enough we can neglect it and just use

$$P=2\pi\sqrt{(L/g)}$$

Using this simpler form allows us to make observations using small angles and use those to compute the period for other pendulum lengths.

Apparatus:

The first apparatus is the stopwatch, shown in figure 1. It consists of an LCD readout which shows time in hundredths of seconds, a 'start/stop' button, and a 'reset' button.

The main apparatus is the pendulum itself, shown in figure 2. It is composed of a fixed connecting point high above the floor on which a string is tied. The other end of the string is tied to a mass which appears to be a lead or steel ball. The mass is free to swing back and forth. The angle of the swing can be viewed on the protractor which is attached in such a way that the base of it is even with the connecting point and the string is crossing the zero degree mark when the mass hangs at rest. The length of the string can be varied by wrapping it up over a second peg above the connecting point. However, to be properly set up the string must wrap clockwise over the connecting point.

The final apparatus is the meter stick, shown in figure 3. It is simply an aluminum stick marked and labeled in centimeters with sub marks in millimeters. It offers measurement up to 100 cm.

Procedure:

Measurements for the first part were made by first setting the length of the pendulum using the string wrapping method mentioned earlier, then carefully measuring the distance from the connecting point to a spot in the center of the mass using the meter stick. The next step was made by having one person pull the mass out to a point where the string indicated an angle of ten degrees on the protractor and released it, after counting down from three. Upon release a second person would press the start/stop button on the stopwatch to start the timer. We would then count ten complete oscillations at which point the person with the stopwatch would press the start/stop button again to stop the timer. We would read the time displayed and divide it by ten to give us the time for one period. After recording the data the stopwatch would be reset to zero with the reset button, and we could then take another measurement. We took five different measurements for each of eight pendulum lengths. We also took 5 measurements for two lengths released from 10 degrees, 30 degrees, and 45 degrees.

The data collected is shown in tables 1 and 2.

table 1	Length of Pendulum (cm)	Period (sec) Trial 1	Period (sec) Trial 2	Period (sec) Trial 3	Period (sec) Trial 4	Period (sec) Trial 5	Mean P (sec)	σP (sec)	Mean P^2 (sec^2)	σP^2 (sec^2)
	24.00	0.954	0.963	0.954	0.944	0.944	0.9518	0.008	0.9059	0.011
	28.50	1.050	1.053	1.059	1.050	1.068	1.0560	0.008	1.1151	0.011
	33.50	1.140	1.128	1.119	1.147	1.134	1.1336	0.011	1.2850	0.017
	38.00	1.244	1.215	1.206	1.218	1.203	1.2172	0.016	1.4816	0.028
	52.50	1.428	1.431	1.434	1.412	1.425	1.4260	0.009	2.0335	0.017
	59.50	1.534	1.541	1.563	1.525	1.500	1.5326	0.023	2.3489	0.050
	72.00	1.672	1.672	1.675	1.675	1.687	1.6762	0.006	2.8096	0.015
	86.50	1.819	1.838	1.835	1.819	1.837	1.8296	0.010	3.3474	0.025

table 2 Angle (°)	Variation of P with angle							
30.00	86.50	1.856	1.887	1.859	1.867	1.869	1.8676	0.012
45.00	86.50	1.913	1.904	1.903	1.903	1.900	1.9046	0.005
30.00	72.00	1.703	1.694	1.700	1.700	1.703	1.7000	0.004
45.00	72.00	1.728	1.735	1.728	1.728	1.719	1.7276	0.006
10.00	86.50	1.819	1.838	1.819	1.819	1.837	1.8264	0.010

The mean periods of oscillation for each pendulum length were calculated using the following formula:

$$\text{Mean } P = 1/N * \Sigma p_i$$

Where N is the total number of data points, p_i are the individual measurements, and $\text{Mean } P$ is the mean value of the period.

The standard deviation given in the tables is calculated by:

$$\sigma_P = \sqrt{(1/(N-1) * \Sigma\ (P_i - \text{Mean } P)^2)}$$

Where σ_P is the standard deviation of the period. The mean value of P^2 is determined by squaring $\text{Mean } P$, and the standard deviation of P^2 , σ_{P^2} , is determined by:

$$\sigma_{P^2} = \sigma_P * \text{Mean } P * \sqrt{2}$$

As indicated on graph 1, I determined the slope to be .0381 ± 0.0005. We can use this to extract an experimentally measured value of g, acceleration due to gravity.
I used slope(m)=4pi^2/g and calculated g to be 1036cm/sec^2 ±13cm/sec^2 which is within 5.7% of actual g of 980 cm/sec^2 which is pretty good considering some of the inaccuracies of our experiment.

Graph 2 shows the relationship between the angle and the period. It shows that it is not only the length but the angle that determines the period. The graph shows a linear relationship with the period increasing as the angle is increased. This makes sense as the mass has to travel a longer distance per period for greater angles.

Errors:

There are many sources for errors in this experiment. The biggest is the inaccurate timing of the periods. There is great random error by having a person starting and stopping the timer. This could be solved by using a photo-timer to accurately measure the periods. Also, there is random error in the measurement of L, as there was no center of mass marked on the mass, and the meter stick was being leisurely held in place to eyeball the measurement. This error can be reduced by using a precise measuring device. The systematic errors we have include air resistance of the mass and string moving through the atmosphere. This could be eliminated in a vacuum chamber, but an experiment in one would not be friendly for new physics students. Also, since the string is wrapping around a peg it travels shorter in one direction than the other, causing inaccurate period readings. This can be solved by affixing the string to a single point to swing from.

Conclusion:

My results were consistent with the small angle theory, and I was able to derive experimentally a fairly accurate value for g.